Chapter 1 • Polynomial Functions

Review of Prerequisite Skills

2. **g.** $(x+n)^2-9$
$=(x+n+3)(x+n-3)$

h. $49u^2-(x-y)^2$
$=(7u+x-y)(7u-x+y)$

i. x^4-16
$=(x^2+4)(x^2-4)$
$=(x^2+4)(x+2)(x-2)$

3. **c.** h^3+h^2+h+1
$=h^2(h+1)+(h+1)$
$=(h+1)(h^2+1)$

e. $4y^2+4yz+z^2-1$
$=(2y+z)^2-1$
$=(2y+z-1)(2y+z+1)$

f. $x^2-y^2+z^2-2xz$
$=x^2-2xz+z^2-y^2$
$=(x-z)^2-y^2$
$=(x-z-y)(x-z+y)$

4. **f.** y^3+y^2-5y-5
$=y^2(y+1)-5(y+1)$
$=(y+1)(y^2-5)$

g. $60y^2+10y-120$
$=10(6y^2-y+12)$
$=10(3y+4)(2y-3)$

5. $36(2x-y)^2-25(u-2y)^2$
$=[6(2x-y)]^2-[5(u-2y)]^2$
$=[6(2x-y)-5(u-2y)][6(2x-y)+5(u-2y)]$
$=[12x-6y-5u+10y][12x-6y+5u-10y]$
$=(12x+4y-5u)(12x-16y+5u)$

c. $y^5-y^4+y^3-y^2+y-1$
$=y^4(y-1)+y^2(y-1)+(y-1)$
$=(y-1)(y^4+y^2+1)$

e. $9(x+2y+z)^2-16(x-2y+z)^2$
$=[3(x+2y+z)-4(x-2y+z)][3(x+2y+z)+4(x-2y+z$
$=[3x+6y+3z-4x+8y-4z][3x+6y+3z+4x-8y+4z]$
$=[-x+14y-z][7x-2y+7z]$

g. $p^2-2p+1-y^2-2yz-z^2$
$=(p-1)^2-(y+z)^2$
$=(p-1+y+z)(p-1-y-z)$

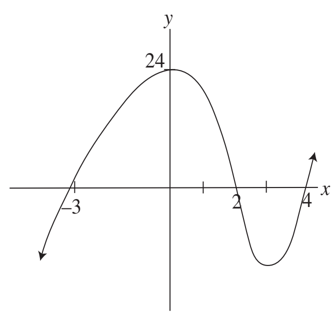
Section 1.1

Investigation 1: Cubic Functions

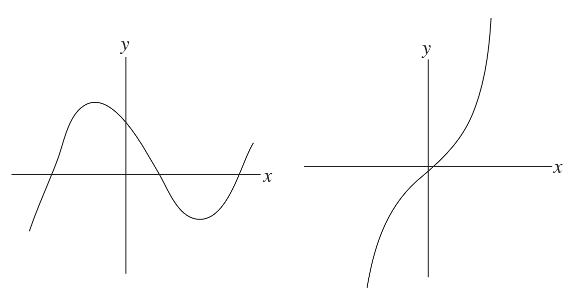
2. There can be 1, or 3 real roots of a cubic equation.

3. **a.** Find the x-intercepts, i.e., the zeros of the function $x = 2$, $x = -3$, $x = 4$, $y = 24$, and the y-intercepts. Since the cubic term has a positive coefficient, start at the lower left, i.e., the third quadrant, crossing the x-axis at –3, then again at 2 and at 4, ending in the upper right of the first quadrant.

b.



4.



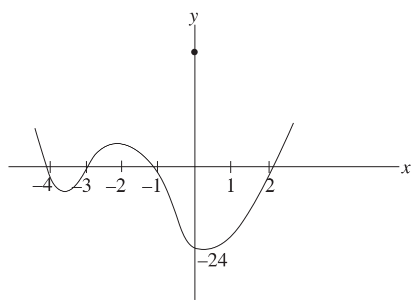
5. When the coefficient of x^3 is negative, the graph moves from the second quadrant to the fourth.

Investigation 2: Quartic Functions

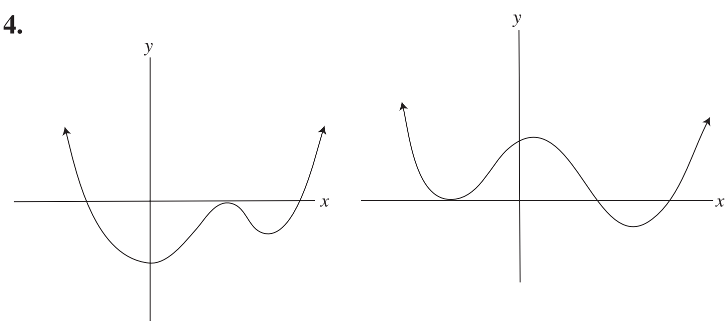
2. There can be 0, 2, or 4 real roots for a quartic equation.

3. **a.** Find the x-intercepts at the function, i.e., $x - 3$, 2, -1, -4. Find the y-intercept, i.e., $y = -24$. Begin in the second quadrant crossing the x-axis at -4, -3, -1, and 2 and end in the first quadrant; draw a smooth curve through intercepts.

b.



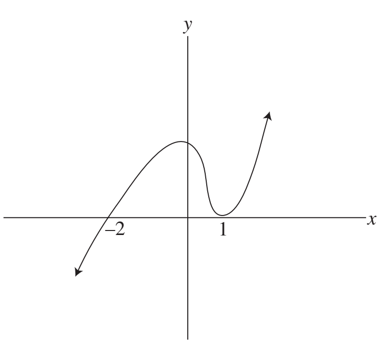
4.



5. If the coefficient of x^4 is negative, the quartic function is a reflection of quartic with a positive coefficient of x^4, i.e. the graph moves from the second to the fourth quadrant, passing through the x-axis a maximum of four times.

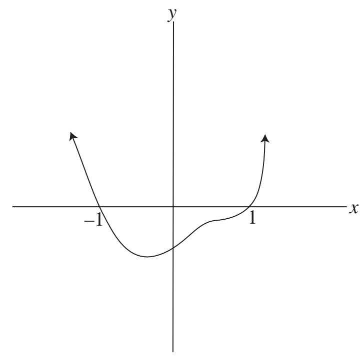
Investigation 3

1.



$$y = (x+2)(x-1)^2$$

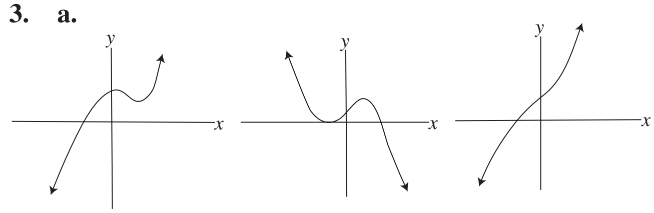
2.



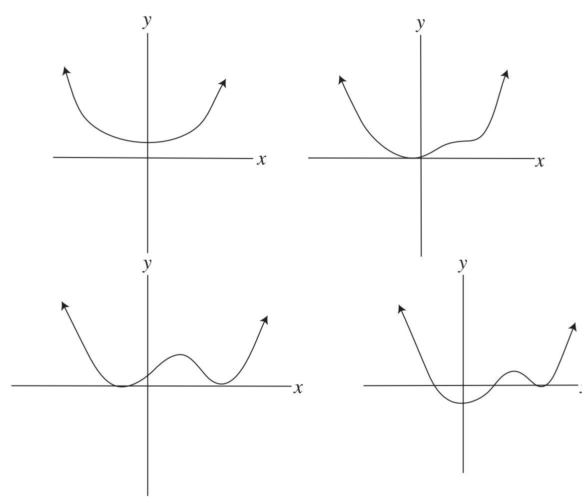
$$y = (x+1)(x-1)^3$$

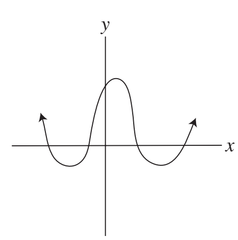
Exercise 1.1

3. **a.**



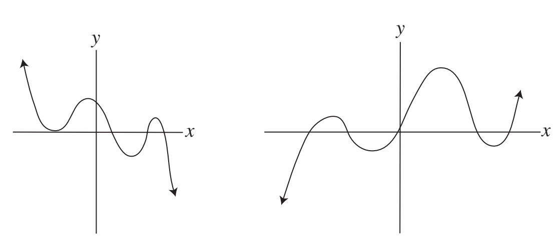
b.



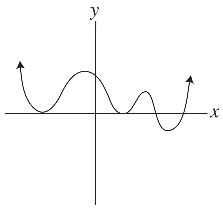


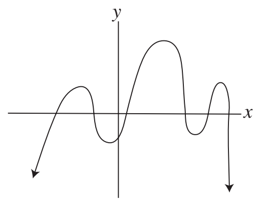
Also includes the reflections of all these graphs in the x-axis.

4. **a.**



b.





Section 1.2

Investigation 1: Cubic Functions

2.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^2 f(x)$
1	1	$8-1=7$	$19-7=12$	$18-12=6$
2	8	$27-8=19$	$37-19=18$	$24-18=6$
3	27	$64-27=37$	$61-37=24$	$30-24=6$
$m-2$	$(m-2)^3$	$(m-1)^3-(m-2)^3=3m^2-9m+7$	$(3m^2-3m+1)-(3m^2-9m+7)=6m-6$	$6m-(6m-6)=6$
$m-1$	$(m-1)^3$	$m^3-(m-1)^3=3m^2-3m+1$	$(3m^2+3m+1)-(3m^2-3m+1)=6m$	$(6m+6)-(6m)=6$
m	m^3	$(m+1)^3-m^3=3m^2+3m+1$	$(3m^2+9m+7)-(3m^2+3m+1)=6m+6$	$(6m+12)-(6m+6)=6$
$m+1$	$(m+1)^3$	$(m+2)^3-(m+1)^3=3m^2+9m+7$	$(3m^2+15m+19)-(3m^2+9m+7)=6m+12$	$(6m+18)-(6m+12)=6$
$m+2$	$(m+2)^3$	$(m+3)^3-(m+2)^3=3m^2+15m+19$	$(3m^2+21m+17)-(3m^2+15m+19)=6m+18$	$(6m+24)-(6m+18)=6$

For quadratic functions, the second finite differences are constant.
For cubic functions, the third finite differences are constant.
It appears that for a polynomial function, a constant finite difference occurs at that difference that is the same as the degree of the polynomial.

Exercise 1.2

1. $(1,0)$, $(2,-2)$, $(3,-2)$, $(4,0)$, $(5,4)$, $(6,10)$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	0	$-2-0=-2$	$0-(-2)=2$
2	-2	$-2-(-2)=0$	$2-0=2$
3	-2	$0-(-2)=2$	$4-2=2$
4	0	$4-0=4$	$6-4=2$
5	4	$10-4=6$	
6	10		

Since $\Delta^2 f(x)$ for any x, then the polynomial function is a quadratic of the form $f(x)=ax^2+bx+c$.

Substituting the given ordered pairs, we get

$f(1)=a+b+c=0$ …(1)

$f(2)=4a+2b+c=-2$ …(2)

$f(3)=9a+3b+c=-2$ …(3)

Solving these equations, we have

$(2)-(1)$ $3a+b=-2$ …(4)

$(3)-(2)$ $5a+b=0$ …(5)

$(5)-(4)$ $2a=2$

$a=1$

Substituting into (4), $3(1)+b=-2$

$b=-5.$

Substituting into (1), $(1)+(-5)+c=0$

$c=4.$

Therefore, the function is $f(x)=x^2-5x+4$.

2. $(1,-1)$, $(2,2)$, $(3,5)$, $(4,8)$, $(5,11)$, $(6,14)$

x	$f(x)$	$\Delta f(x)$
1	-1	$2-(-1)=3$
2	2	$5-2=3$
3	5	$8-5=3$
4	8	$11-8=3$
5	11	$14-11=3$
6	14	$\vdots$

Since $\Delta f(x) = 3$ for any x, then the function is linear of the form $y = mx + b$.

Substituting the given ordered pairs, we get

$f(1)=m+b=-1$ $\quad\ldots(1)$

$f(2)=2m+b=2$ $\quad\ldots(2).$

Solving these equations, we get

$(2)-(1)$ $\quad m=3.$

Substituting into $\ldots(1)$

$3+b=-1$

$b=-4.$

Therefore, the function is $f(x)=3x-4$.

3. $(1,4)$, $(2,15)$, $(3,30)$, $(4,49)$, $(5,72)$, $(6,99)$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	4	$15-4=11$	$15-11=4$
2	15	$30-15=15$	$19-15=4$
3	30	$49-30=19$	$23-19=4$
4	49	$72-49=23$	$27-23=4$
5	72	$99-72=27$	
6	99		

Since $\Delta^2 f(x)$ is constant, the function is of the form $f(x)=ax^2+bx+c$.

Substituting the given ordered pairs,

$f(1)=a+b+c=4$ $\quad\ldots(1)$

$f(2)=4a+2b+c=15$ $\quad\ldots(2)$

$f(3)=9a+3b+c=30$ $\quad\ldots(3).$

Solving these equations, we have

$(2)-(1)$ $\quad 3a+b=11$ $\quad\ldots(4)$

$(3)-(2)$ $\quad 5a+b=15$ $\quad\ldots(5)$

$(5)-(4)$ $\quad 2a=4$

$a=2.$

Substituting into (5) $\quad 5(2)+b=15$

$b=5.$

Substituting into (1) $\quad 2+5+c=4$

$c=-3.$

Therefore, the function is $f(x)=2x^2+5x-3$.

4. $(1, -9)$, $(2, -10)$, $(3, -7)$, $(4, 0)$, $(5, 11)$, $(6, 26)$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	-9	$-10-(-9)=1$	$3-(-1)=4$
2	-10	$-7-(-10)=3$	$7-3=4$
3	-7	$0-(-7)=7$	$11-7=4$
4	0	$11-0=11$	$15-11=4$
5	11	$26-11=15$	
6	26		

Since $\Delta^2 f(x)$ is constant, the function is of the form $f(x) = ax^2 + bx + c$.

Substituting the given ordered pairs,

$f(1)=a+b+c=0$ $\quad\ldots(1)$

$f(2)=4a+2b+c=-2$ $\quad\ldots(2)$

$f(3)=9a+3b+c=-2$ $\quad\ldots(3).$

Solving these equations,

$(2)-(1)$ $\quad 3a+b=-2$ $\quad\ldots(4)$

$(3)-(2)$ $\quad 5a+b=0$ $\quad\ldots(5)$

$(5)-(4)$ $\quad 2a=2$

$a=1.$

Substituting into (4), $\quad 3(1)+b=-2$

$b=-5.$

Substituting into (1), $\quad (1)+(-5)+c=0$
$$c=4.$$

Therefore, the function is $f(x)=x^2-5x+4$.

5. $(1,\ 12),\ (2,\ -10),\ (3,\ -18),\ (4,\ 0),\ (5,\ 56),\ (6,\ 162)$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	12	$(-10)-(-12)=-22$	$-8-(-22)=14$	$26-14=12$
2	-10	$(-18)-(-10)=-8$	$18-(-8)=26$	$38-26=12$
3	-18	$0-(-18)=18$	$56-18=38$	$50-38=12$
4	0	$56-0=56$	$106-56=50$	
5	56	$162-56=106$		
6	162			

Since $\Delta f(x)$ is constant, the function is of the form $f(x)=ax^3+bx^2+cx+d$.

Substituting the given ordered pairs,

$f(1)=a+b+c+d=12 \quad \ldots(1)$

$f(2)=8a+4b+2c+d=-10 \quad \ldots(2)$

$f(3)=27a+9b+3c+d=-18 \quad \ldots(3)$

$f(4)=64a+16b+4c+d=0 \quad \ldots(4).$

Solving the equations,

$(2)-(1) \quad 7a+3b+c=-22 \quad \ldots(5)$

$(3)-(2) \quad 19a+5b+c=-8 \quad \ldots(6)$

$(4)-(3) \quad 37a+7b+c=18 \quad \ldots(7)$

$(6)-(5) \quad 12a+2b=14 \quad \ldots(8)$

$(7)-(6) \quad 18a+2b=26 \quad \ldots(9)$

$(9)-(8) \quad 6a=12$
$$a=2.$$

Substituting into (8), $\quad 12(2)+2b=14$
$$b=-5.$$

Substituting into (5), $\quad 7(2)+3(-5)+c=-23$
$$c=-21.$$

Substituting into (1), $\quad 2-5-21+d=12$
$$d=36.$$

Therefore, the function is
$f(x)=2x^3-5x^2-21x+36$.

6. $(1,-34),\ (2,-42),\ (3,-38),\ (4,-16),\ (5,30),\ (6,106)$
Using differences we obtain the following.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	-34	-8	12	6
2	-42	4	18	6
3	-38	22	24	6
4	-16	46	30	$\vdots$
5	30	76	$\vdots$	$\vdots$
6	106	$\vdots$	$\vdots$	$\vdots$

Since is $\Delta^3 f(x)$ is constant, the function is of the form $f(x) = ax^3 + bx^2 + cx + d$.

Substituting the given ordered pairs,

$f(1)=a+b+c+d=-34 \quad \ldots(1)$

$f(2)=8a+4b+2c+d=-42 \quad \ldots(2)$

$f(3)=27a+9b+3c+d=-38 \quad \ldots(3)$

$f(4)=64a+16b+4c+d=-16 \quad \ldots(4).$

Solving the equations,

$(2)-(1) \quad 7a+3b+c=-8 \quad \ldots(5)$

$(3)-(2) \quad 19a+5b+c=4 \quad \ldots(6)$

$(4)-(3) \quad 37a+7b+c=22 \quad \ldots(7)$

$(6)-(5) \quad 12a+2b=12 \quad \ldots(8)$

$(7)-(6) \quad 18a+2b=18 \quad \ldots(9)$

$(9)-(8) \quad 6a=6$
$$a=1.$$

Substituting into (8), $\quad 12(1)+2b=12$
$$b=0.$$

Substituting into (5), $\quad 7(1)+3(0)+c=-18$
$$c=-15.$$

Substituting into (1), $\quad 1+0-15+d=-34$
$$d=-20.$$

Therefore, the function is $f(x)=x^3-15x-20$.

7. $(1, 10), (2, 0), (3, 0), (4, 16), (5, 54), (6, 120), (7, 220)$

Using differences, we obtain the following.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	10	– 10	10	6
2	0	0	16	6
3	0	16	22	6
4	16	38	28	
5	54	66	34	
6	120	100		
7	220			

Since $\Delta^3 f(x)$ is constant, the function is of the form $f(x) = ax^3 + bx^2 + cx + d$.

Substituting the given ordered pairs,

$f(1) = a + b + c + d = 10$ …(1)

$f(2) = 8a + 4b + 2c + d = 0$ …(2)

$f(3) = 27a + 9b + 3c + d = 0$ …(3)

$f(4) = 64a + 16b + 4c + d = 16$ …(4).

Solving the equations,

$(2)-(1)$ $7a + 3b + c = -10$ …(5)

$(3)-(2)$ $19a + 5b + c = 0$ …(6)

$(4)-(3)$ $37a + 7b + c = 16$ …(7)

$(6)-(5)$ $12a + 2b = 10$ …(8)

$(7)-(6)$ $18a + 2b = 16$ …(9)

$(9)-(8)$ $6a = 6$

$a = 1.$

Substituting into (8), $12(1) + 2b = 10$

$b = -1.$

Substituting into (5), $7(1) + 3(-1) + c = -10$

$c = -14.$

Substituting into (1), $1 - 1 - 14 + d = 10$

$d = 24.$

Therefore, the function is $f(x) = x^3 - x^2 - 14x + 24$.

8. $(1, -4), (2, 0), (3, 30), (4, 98), (5, 216), (6, 396)$

Using differences, we obtain the following.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	– 4	4	26	12
2	0	30	38	12
3	30	68	50	12
4	98	118	62	
5	216	180		
6	396			

Since $\Delta^3 f(x)$ is constant, the function is of the form $f(x) = ax^3 + bx^2 + cx + d$.

Substituting the given ordered pairs,

$f(1) = a + b + c + d = -4$ …(1)

$f(2) = 8a + 4b + 2c + d = 0$ …(2)

$f(3) = 27a + 9b + 3c + d = 30$ …(3)

$f(4) = 64a + 16b + 4c + d = 98$ …(4).

Solving the equations,

$(2)-(1)$ $7a + 3b + c = 4$ …(5)

$(3)-(2)$ $19a + 5b + c = 30$ …(6)

$(4)-(3)$ $37a + 7b + c = 68$ …(7)

$(6)-(5)$ $12a + 2b = 26$ …(8)

$(7)-(6)$ $18a + 2b = 38$ …(9)

$(9)-(8)$ $6a = 12$

$a = 2.$

Substituting into (8), $12(2) + 2b = 26$

$b = 1.$

Substituting into (5), $7(2) + 3(1) + c = 4$

$c = -13.$

Substituting into (1), $2 + 1 - 13 + d = -4$

$d = 6.$

Therefore, the function is $f(x) = 2x^3 + x^2 - 13x + 6$.

9. $(1,-2),\ (2,-4),\ (3,-6),\ (4,-8),\ (5,14),\ (6,108),\ (7,346)$

Using differences, we obtain the following:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	– 2	– 2	0	0	24
2	– 4	– 2	0	24	24
3	– 6	– 2	24	48	24
4	– 8	22	72	72	
5	14	94	144		
6	108	238			
7	346				

Since $\Delta^4 f(x)$ is constant, the function is of the form $f(x) = ax^4 + bx^3 + dx + e$.

Substituting the given ordered pairs,

$f(1) = a + b + c + d + e = -2$ …(1)

$f(2) = 16a + 8b + 4c + 2d + e = -4$ …(2)

$f(3) = 81a + 27b + 9c + 3d + e = -6$ …(3)

$f(4) = 256a + 64b + 16c + 4d + e = -8$ …(4).

Solving the equations,

$(2)-(1)$ $15a + 7b + 3c + d = -2$ …(6)

$(3)-(2)$ $65a + 19b + 5c + d = -2$ …(7)

$(4)-(3)$ $175a + 37b + 7c + d = -2$ …(8)

$(5)-(4)$ $369a + 61b + 9c + d = 22$ …(9)

$(7)-(6)$ $50a + 12b + 2c = 0$ …(10)

$(8)-(7)$ $110a + 18b + 2c = 0$ …(11)

$(9)-(8)$ $194a + 24b + 2c = 24$ …(12)

$(11)-(10)$ $60a + 6b = 0$ …(13)

$(12)-(11)$ $84a + 6b = 24$ …(14)

$(14)-(13)$ $24a = 24$

$a = 1.$

Substituting into (13), $60(1) + 6b = 0$

$b = -10.$

Substituting into (10), $50(1) + 12(-10) + 2c = 0$

$c = 35.$

Substituting into (6), $15(1) + 7(-10) + 3(35) + d = -2$

$d = -52.$

Substituting into (1), $1 - 10 + 35 - 52 + e = -2$

$e = 24.$

Therefore, the function is

$f(x) = x^4 - 10x^3 + 35x^2 - 52x + 24.$

10. $(1,1),\ (2,2),\ (3,4),\ (4,8),\ (5,16),\ (6,32),\ (7,64)$

Using differences, we obtain the following:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
1	1	1	1	1	1	1
2	2	2	2	2	2	2
3	4	4	4	4	4	4
4	8	8	8	8	8	
5	16	16	16	16		
6	32	32	32			
7	64					

As there is no constant difference, this will not be defined as a polynomial function. This is $f(x) = 2^{n-1}$, by inspection.

11. a. Using the $\boxed{\text{STAT}}$ function, the function is $V = -0.0374x^3 + 0.1522x^2 + 0.1729x.$

b. The maximum volume of air during the cycle is 0.8863 and occurs after 3.2.

12. a.

t	$f(t)$	$\Delta f(t)$	$\Delta^2 f(t)$	$\Delta^3 f(t)$
1	4031	–23	–48	6
2	4008	–71	–42	6
3	3937	–113	–36	
4	3824	–149		
5	3675	–179		
6	3496			

Since the third differences are constant, it forms a cubic function. Using the $\boxed{\text{STAT}}$ mode on the graphing calculator, $f(t) = t^3 - 30t^2 + 60t + 4000.$

b. From the graph of $f(t)$, it seems that the population began to increase 9 years ago, in 1971.

c. For the year 2030, $t = 50$.

$$\begin{aligned} f(50) &= 50^3 - 30(50)^2 + 60(50) + 4000 \\ &= 57\ 000 \end{aligned}$$

So, if the function continues to describe the population after 2002, in the year 2030, it will be about 57 000.

Exercise 1.3

7. b.

$$\begin{array}{r} x^2 + 5x + 2 \\ x-1\overline{)\,x^3 + 4x^2 - 3x - 2} \\ \underline{x^3 - x^2 \qquad\qquad} \\ 5x^2 - 3x \quad\; \\ \underline{5x^2 - 5x \quad\;} \\ 2x - 2 \\ \underline{2x - 2} \\ 0 \end{array}$$

Since the remainder is 0, $x - 1$ is a factor of $x^3 + 4x^2 - 3x - 2$. The other factor is $x^2 + 5x + 2$.
$x^3 + 4x^2 - 3x - 2 = (x-1)(x^2 + 5x + 2)$

c.

$$\begin{array}{r} 2x^2 + 2x + 3 \\ x-3\overline{)\,2x^3 - 4x^2 - 3x + 5} \\ \underline{2x^3 - 6x^2 \qquad\qquad} \\ 2x^2 - 3x \quad\; \\ \underline{2x^2 - 6x \quad\;} \\ 3x + 5 \\ \underline{3x - 9} \\ 14 \end{array}$$

Since the remainder $r(x) = 14$ is of a degree less than that of the divisor, the division is complete. So, $2x^3 - 4x^2 - 3x + 5 = (x-3)(2x^2 + 2x + 3) + 14$.

g.

$$\begin{array}{r} 2x^2 - 3 \\ 2x+3\overline{)\,4x^3 + 6x^2 - 6x - 9} \\ \underline{4x^3 + 6x^2 \qquad\qquad} \\ -6x - 9 \\ \underline{-6x - 9} \\ 0 \end{array}$$

Since the remainder is 0, $2x + 3$ is a factor of $4x^3 + 6x^2 - 6x - 9$. Therefore,
$4x^3 + 6x^2 - 6x - 9 = (2x+3)(2x^2 - 3)$.

h.

$$\begin{array}{r} x^2 - 3x + 5 \\ 3x-2\overline{)\,3x^3 - 11x^2 + 21x - 7} \\ \underline{3x^3 - 2x^2 \qquad\qquad\quad} \\ -9x^2 + 21x \quad\; \\ \underline{-9x^2 + 6x \quad\;} \\ 15x - 7 \\ \underline{15x - 10} \\ +3 \end{array}$$

Since the remainder, $r(x) = 3$ is of a degree less than that of the divisor, the division is complete. So, $3x^3 - 11x^2 + 21x - 7 = (3x-2)(x^2 - 3x + 5) + 3$.

9. b.

$$\begin{array}{r} 2x^3 - 2x^2 - x + 1 \\ x+1\overline{)\,2x^4 + 0x - 3x^2 + 1} \\ \underline{2x^4 + 2x^3 \qquad\qquad\quad} \\ -2x^3 - 3x^2 \qquad\quad \\ \underline{-2x^3 - 2x^2 \qquad\quad} \\ -x^2 \qquad\quad \\ \underline{-x^2 - x \quad\;\;} \\ x + 1 \\ \underline{x + 1} \\ 0 \end{array}$$

c.

$$
\begin{array}{r}
4x^2 - 8x + 16 \\
x+2\overline{\big)\,4x^3 + 0x^2 + 0x + 3^2 + 32} \\
\underline{4x^3 + 8x^2} \\
-8x^2 \\
\underline{-8x^2 - 16x} \\
16x + 32 \\
\underline{16x + 32} \\
0
\end{array}
$$

d.

$$
\begin{array}{r}
x^4 + x^3 + x^2 + x + 1 \\
x-1\overline{\big)\,x^5 + 0x^4 + 0x^3 + 0x^2 + 0x^1 + 0x - 1} \\
\underline{x^5 - x^4} \\
x^4 \\
\underline{x^4 - x^3} \\
x^3 \\
\underline{x^3 - x^2} \\
x^2 \\
\underline{x^2 - x} \\
x - 1 \\
\underline{x - 1} \\
0
\end{array}
$$

12. Dividing *f(x)* by *d(x)*.

$$
\begin{array}{r}
x^2 - x \\
x^2 + 2x + 1\overline{\big)\,x^4 + x^3 - x^2 - x} \\
\underline{x^4 + 2x^3 + x^2} \\
-x^3 - 2x^2 - x \\
\underline{-x^3 - 2x^2 - x} \\
0
\end{array}
$$

16. $x = yq + r$ where $y \le x$ and $x, y \in N$

a. If y is a factor of x, it will divide into x without leaving a remainder. So, $r = 0$.

b. The value of the remainder must be less than that of the divisor if the division is complete, and y is not a factor of $9x$, so if $y = 5$, the values of r are 1, 2, 3, or 4. If $y = 7$, $r = 1, 2, 3, 4, 5, 6$, and if $r = n$, $r = 1, 2, 3, \ldots, n - 1$.

17. a.

$$
\begin{array}{r}
x^2 + 6x + 7 \\
x-2\overline{\big)\,x^3 + 4x^2 - 5x - 9} \\
\underline{x^3 - 2x^2} \\
6x^2 - 5x \\
\underline{6x^2 - 12x} \\
7x - 9 \\
\underline{7x - 14} \\
5
\end{array}
$$

So, $x^3 + 4x^2 - 5x - 9 = (x-2)(x^2 + 6x + 7) + 5$ where $q(x) = x^2 + 6x + 7$ and $r = 5$.

$$
\begin{array}{r}
x + 5 \\
x+1\overline{\big)\,x^2 + 6x + 7} \\
\underline{x^2 + x} \\
5x + 7 \\
\underline{5x + 5} \\
2
\end{array}
$$

So, $x^2 + 6x + 7 = (x+1)(x+5) + 2$, where $Q(x) = x + 5$ and $r_2 = 2$.

b. If $f(x)$ is divided by $(x-2)(x+1)$, the quotient is the $Q(x)$ obtained in **a.** Since
$x^3 + 4x^2 - 5x - 9 = (x-2)(x^2 + 6x + 7) = 5,$
by substituting,

$$= (x-2)\big[(x+1)(x+5) + 2\big] + 5$$
$$= (x-2)\big[(x+1)(x+5)\big] + (x-2)\big[(2)\big] + 5$$

and simplifying,

$$= (x-2)(x+1)(x+5) + 2(x-2) + 5$$
$$= (x-2)(x+1)(x+5) + 2x + 1.$$

Therefore, when *f(x)* is divided by $(x-2)(x+1)$, the quotient is $(x+5)$ and the remainder is $2(x-2) + 5$ or $2x + 1$.

Exercise 1.4

2. **b.** When $f(x)$ is divided by $x+1$, the remainder is $f(-1)$.

$$r = f(-1)$$
$$= (-1)^3 - 4(-1)^2 + 2(-1) - 6$$
$$= -13$$

c. When $f(x)$ is divided by $2x-1$, the remainder is $f\left(\frac{1}{2}\right)$.

$$r = f\left(\frac{1}{2}\right)$$
$$= \left(\frac{1}{2}\right)^3 - 4\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right) - 6$$
$$= -\frac{47}{8} \text{ or } -5.875$$

d. When $f(x)$ is divided by $2x+3$, the remainder is $f\left(-\frac{3}{2}\right)$.

$$r = f\left(-\frac{3}{2}\right)$$
$$= \left(-\frac{3}{2}\right)^3 - 4\left(-\frac{3}{2}\right)^2 + 2\left(-\frac{3}{2}\right) - 6$$
$$= -\frac{171}{8} \text{ or } -21.375$$

3. **c.** Let $f(x) = 2x^3 + 4x - 1$.

The remainder when is divided by $x+2$ is

$$r = f(-2)$$
$$= 2(-2)^3 + 4(-2) - 1$$
$$= -25.$$

f. Let $f(x) = -2x^4 + 3x^2 - x + 2$.

When $f(x)$ is divided by $x+2$, the remainder is

$$r = f(-2)$$
$$= -2(-2)^4 + 3(-2)^2 - (-2) + 2$$
$$= -2(16) + 3(4) + 2 + 2$$
$$= -16.$$

4. **f.** The remainder is

$$r = f\left(\frac{1}{2}\right)$$
$$= 4\left(\frac{1}{2}\right)^3 + 9\left(\frac{1}{2}\right) - 10$$
$$= -5.$$

5. **a.** Since the remainder is 1 when the divisor is $x+2$, then $f(-2) = 1$ by the Remainder Theorem.

$$(-2)^3 + k(-2)^2 + 2(-2) - 3 = 1$$
$$-8 + 4k - 4 - 3 = 1$$
$$4k = 16$$

b. Since the remainder is 16 when the divisor is $x-3$, then $f(3) = 16$ by the Remainder Theorem.

$$(3)^4 - k(3)^3 - 2(3)^2 + (3) + 4 = 16$$
$$-27k = -54$$
$$k = 2$$

c. Since the remainder is 1 when the divisor is $2x-1$, then

$f\left(\frac{1}{2}\right) = 1$ by the Remainder Theorem.

$$2\left(\frac{1}{2}\right)^3 - 3\left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - 1 = 1$$
$$\frac{1}{4} - \frac{3}{4} + \frac{1}{2}k - 1 = 1$$
$$\frac{1}{2}k = \frac{5}{2}$$
$$k = 5$$

6. $f(x) = mx^3 + gx^2 - x + 3$

When the divisor is $x+1$, the remainder is 3.

By the Remainder Theorem, $f(-1) = 3$

$$m(-1)^3 + g(-1)^2 - (-1) + 3 = 3$$
$$-m + g = -1. \qquad (1)$$

When the divisor is $x+2$, the remainder is -7.

Therefore, $f(-2) = -7$

$$m(-2)^3 + g(-2)^2 - (-2) + 3 = -7$$
$$-8m + 4g = -12$$
$$\text{or} \qquad 2m - g = 3. \qquad (2)$$

Solving the resulting linear equations,

$(2)+(1) \qquad m=2.$

Substituting into (1), $\quad g=1.$

7. $f(x)=mx^3+gx^2-x+3$

When the divisor is $x-1$, the remainder is 3.

By the Remainder Theorem, $f(1)=3$.

$$m+g-1+3=3$$
$$m+g=1 \qquad (1)$$

When the divisor is $x+3$, the remainder is -1.

So, $f(-3)=-1$.

$$m(-3)^3+g(-3)^2-(-3)+3=-1$$
$$-27m+9g=-7 \qquad (2)$$

$$9\times(1) \qquad \underline{9m+9g=9}$$
$$-36m=-16$$
$$m=\frac{4}{9}$$

Substituting into (1), $\quad g=\frac{5}{9}.$

8. *Solution 1: Using the Remainder Theorem*

Let $f(x)=x^3+3x^2-x-2. \qquad (1)$

Then, $f(x)=(x+3)(x+5)q(x)+r(x)$

where x is a linear expression.

Let $r(x)=Ax+B.$

So, $f(x)=(x+3)(x+5)q(x)+(Ax+B). \qquad (2)$

From (2), $f(-3)=(0)(2)q(x)+(-3A+B)$
$=-3A+B.$

From (1), $f(-3)=(-3)^3+3(-3)^2-(-3)-2$
$=1.$

Therefore, $-3A+B=1. \qquad (3)$

Similarly, $f(-5)=(-2)(0)q(x)+(-5A+B)$
$=-5A+B$

and $f(-5)=(-5)^3+3(-5)^2-(-5)-2$
$=-47.$

Therefore, $-5A+B=-47. \qquad (4)$

Solving (3) and (4),

$(3)-(4) \qquad 2A=48$
$A=24.$

Substituting in (4), $B=73$.

Since $r(x)=Ax+B$
$=24x+73.$

The remainder is $24+73$.

Solution 2: Using Long Division

Expanding $(x+3)(x+5)=x^2+8x+15$

$$\begin{array}{r} x-5 \\ x^2+8x+15\overline{)\,x^3+3x^2-x-2} \\ \underline{x^3+8x+15x} \\ -5x^2-16x-2 \\ \underline{-5x^2-40x-75} \\ 24x+73 \end{array}$$

The remainder is $24x+73$.

9. *Solution 1: Using Long Division*

Expanding the divisor $(x-1)(x+2)=x^2+x-2$

$$\begin{array}{r} 3x^3-3x^2+9x-20 \\ x^2+x-2\overline{)\,3x^5-5x^2+4x+1} \\ \underline{3x^5+3x^4-6x^3} \\ -3x^4+6x^3-5x^2 \\ \underline{-3x^4-3x^3+6x^2} \\ 9x^3-11x^2+4x \\ \underline{9x^3+9x^2-18x} \\ -20x^2+22x+1 \\ \underline{-20x^2-20x+40} \\ 42x-39 \end{array}$$

The remainder is $42x-39$.

Solution 2: Using the Remainder Theorem

Let $f(x)=3x^5-5x^2+4x+1.$ (1)

So, $f(x)=(x-1)(x+2)q(x)+Ax+B$ (2)

since $r(x)$ is at most a linear expression.

Since $f(1)=0(3)q(x)+A+B$ from (1)

and $f(1)=3(1)^5-5(1)^2+4(1)+1$ from (2)

$=3.$

So $A+B=3.$ (3)

Similarly, $f(-2)=(-3)(0)q(x)+A(-2)+B$ from (1)

and $f(-2)=3(-2)^5-5(-2)^2+4(-2)+1$ from (2)

So $-2A+B=-123.$

Solving (3) and (4) by subtracting,

and $3A=126$

$A=42$

$B=-39.$

The remainder is $42x-39.$

10. If the remainder is 3 when $x+2$ is divided into $f(x)$, then $f(-2)=3$.

a. Since the remainder is a constant, adding 1 to $f(x)$, increases the remainder by 1. So, the remainder is $3+1=4$.

b. Since $(x+2)$ is divisible exactly by the divisor $x+2$, there is no remainder for that division. So, the remainder for $f(x)+x+2$ is the same as that for $f(x)$, i.e., the remainder is 3.

c. The remainder of $f(x)$ divided by $x+2$ is 3. By the Remainder Theorem, the remainder of $(4x+7)$ divided by $x+2$ is $4(-2)+7=-1$.
Therefore, the remainder of $f(x)+4x+7$ is the remainder of $f(x)$ plus the remainder of $4x+7$, that is, $3-1=2.$

d. The remainder of $f(x)$ divided by $x+2$ is 3. Hence, the remainder of $2f(x)$ divided by $x+2$ is $2(3)=6.$ The remainder of -7 divided by $x+2$ is -7. So, the remainder of $2f(x)-7$ is $6-7=-1$.

e. If $[f(x)]^2$ is divided by $x+2$, the division statement becomes $f(x)f(x)=(x+2)q(x)+r.$

Let $x=-2,$ then $f(-2)f(-2)=0q(x)+r$

$(3)(3)=r$

$9=r.$

The remainder is 9.

11. In order to have a multiple of $(x+5)$, there must be no remainder after division by $x+5$. The remainder for $f(x)$ is $x+3$. The first multiple for the remainder is $x+5$, or $(x+3)+2$. So, the first multiple greater than $f(x)$ is $f(x)+2$.

12. Factoring by completing a square:

a.
$$\begin{aligned}&x^4+5x+9\\ &=x^4+6x^2+9-x^2\\ &=(x^2+3)^2-x^2\\ &=(x^2+3+x)(x^2+3-x)\\ &=(x^2+x+3)(x^2-x-3)\end{aligned}$$

b.
$$\begin{aligned}&9y^4+8y^2+4\\ &=9y^4+12y^2+4-4y^2\\ &=(3y^2+2)^2-4y^2\\ &=(3y^2+2y+2)(3y^2-2y+2)\end{aligned}$$

c.
$$\begin{aligned}&x^4+6x^2+25\\ &=x^4+10x^2+25-4x^2\\ &=(x^2+5)^2-4x^2\\ &=(x^2+2x+5)(x^2-2x+5)\end{aligned}$$

d.
$$\begin{aligned}&4x^4+8x^2+9\\ &=4x^4+12x^2+9-4x^2\\ &=(2x^2+3)^2-4x^2\\ &=(2x^2+2x+3)(2x^2-2x+3).\end{aligned}$$

Review Exercise

2. a.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
–1	–27	16	– 10	6
0	–11	6	– 4	6
1	–5	2	2	6
2	–3	4	8	
3	1	12		
4	13			

Since $\Delta^3 f(x)$ is constant, $f(x)$ is of the form $f(x) = ax^3 + bx^2 = cx + d.$

Substituting the given ordered pairs,

$f(0) = d = -11$

$f(1) = a + b + c + d = -5.$

Substituting for d, $a + b + c = 6$ (1)

$f(2) = 8a + 4b + 2c + d = -3$

$8a + 4b + 2c = 8$

$4a + 2b + c = 4$ (2)

$f(3) = 27a + 9b + 3c + d = 1$

$27a + 9b + 3c = 12$

$9a + 3b + c = 4$ (3)

Solving,

(2) – (1) $3a + b = -2$ (4)

(3) – (2) $5a + b = 0$ (5)

(5) – (4) $-2a = -2$

$a = 1.$

Substituting into (5), $b = -5$.
Substituting into (1), $c = 10$.

Therefore, the function is
$f(x) = x^3 - 5x^2 + 10x - 11.$

b.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
0	4	11	6	12
1	15	17	18	12
2	32	35	30	12
3	67	65	42	
4	132	107		
5	239			

Since is $\Delta^3 f(x)$ constant, $f(x)$ is of the form $f(x) = ax^3 + bx^2 + cx + d.$

Substituting the given ordered pairs,

$f(0) = d = 4$

$f(1) = a + b + c + 4 = 15$

$a + b + c = 11$ (1)

$f(2) = 8a + 4b + 2c + 4 = 32$

$8a + 4b + 2c = 28$

$4a + 2b + c = 14$ (2)

$f(3) = 27a + 9b + 3c + 4 = 67$

$27a + 9b + 3c = 63$

$9a + 3b + c = 21$ (3)

Solving

(2) – (1) $3a + b = 3$ (4)

(3) – (2) $5a + b = 7$ (5)

(5) – (4) $2a = 4$

$a = 2$

Substituting into (5), $b = -3$.
Substituting into (1), $c = 12$.
Therefore, the function is $f(x) = 2x^3 - 3x^2 + 12x + 4$.

c.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
1	– 9	– 22	22	60	24
2	– 31	0	82	84	24
3	– 31	82	166	108	
4	51	248	274		
5	299	522			
6	821				

Since $\Delta^4 f(x)$ is constant,
$f(x) = ax^4 + bx^3 + cx^2 + dx + e.$
Substituting the given ordered pairs,
$f(1) = a + b + c + d + e = -9$ (1)
$f(2) = 16a + 8b + 4c + 2d + 2 = -31$ (2)
$f(3) = 81a + 27b + 9c + 3d + e = -31$ (3)
$f(4) = 256a + 64b + 16c + 4d + e = 51$ (4)
$f(5) = 625a + 125b + 25c + 5d + e = 299$ (5)

Solving the equations,

(2) – (1)	$15a + 7b + 3c + d = -22$	(6)
(3) – (2)	$65a + 19b + 5c + d = 0$	(7)
(4) – (3)	$175a + 37b + 7c + d = 82$	(8)
(5) – (4)	$369a + 61b + 9c + d = 228$	(9)
(7) – (6)	$50a + 12b + 2c = 22$	(10)
(8) – (7)	$110a + 18b + 2c = 0$	(11)
(9) – (8)	$194a + 24b + 2c = 166$	(12)
(11) – (10)	$60a + 6b = 60$	(13)
(12) – (11)	$84a + 6b = 84$	(14)
(14) – (13)	$24a = 24$	
	$a = 1.$	

Substituting, $b = 0$

$c = -14$

$d = 5$

$e = -1.$

Therefore, $f(x) = x^4 - 14x^2 + 5x - 1.$

d.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
1	1	1	2	6
2	2	3	8	
3	5	11		
4	16			

There is not enough information to find a constant finite difference.

e.

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$
–2	75	–86	76	–72
–1	–11	–10	4	24
0	–21	–6	–20	
1	–27	–26		
2	–53			

There is not enough information to establish the function.

3. c.

$$
\begin{array}{r|l}
 & 2x^2 + 3x - 2 \\
2x + 1 & 4x^3 + 8x^2 - x + 1 \\
 & \underline{4x^3 + 2x^2} \\
 & \quad 6x^2 - x \\
 & \quad \underline{6x^2 + 3x} \\
 & \qquad -4x + 1 \\
 & \qquad \underline{-4x - 2} \\
 & \qquad\qquad 3
\end{array}
$$

$$4x^3 + 8x^2 - x + 1 = (2x + 1)(2x^2 + 3 - 2) + 3$$

d.

$$
\begin{array}{r|l}
 & x^2 - 5x + 10 \\
x^2 + x - 2 & x^4 - 4x^3 + 3x^2 - 3 \\
 & \underline{x^4 + x^3 - 2x^2} \\
 & \quad -5x^3 + 5x^2 \\
 & \quad \underline{-5x^3 - 5x^2 + 10x} \\
 & \qquad 10x^2 - 10x - 3 \\
 & \qquad \underline{10x^2 + 10x - 20} \\
 & \qquad\quad -20x + 17
\end{array}
$$

$$x^4 - 4x^3 + 3x^2 - 3 = (x^2 + x - 2)(x^2 - 5x + 10) - 20x + 17$$

4. c. Let $f(x) = x^3 - 5x^2 + 2x - 1$.

The remainder is

$$
\begin{aligned}
f(-2) &= (-2)^3 - 5(-2)^2 + 2(-2) - 1 \\
&= -8 - 20 - 4 - 1 \\
&= -33.
\end{aligned}
$$

e. Let $f(x) = 3x^3 + x + 2.$

The remainder is
$$
\begin{aligned}
f\left(\frac{1}{3}\right) &= 3\left(\frac{1}{3}\right)^3 + \frac{1}{3} + 2 \\
&= \frac{22}{9}.
\end{aligned}
$$

5. a.

$$
\begin{array}{r|l}
 & x^2 + 3x + 2 \\
x - 1 & x^3 + 2x^2 - x - 2 \\
 & \underline{x^3 - x^2} \\
 & \quad 3x^2 - x \\
 & \quad \underline{3x^2 - 3x} \\
 & \qquad 2x - 2 \\
 & \qquad \underline{2x - 2} \\
 & \qquad\quad 0
\end{array}
$$

$$
\begin{aligned}
x^3 + 2x^2 - x - 2 &= (x - 1)(x^2 + 3x + 2) \\
&= (x - 1)(x + 1)(x + 2)
\end{aligned}
$$

c.

$$
\begin{array}{r}
3x^2 + 11x - 4 \\
2x + 3 \overline{\smash{\big)}\, 6x^3 + 31x^2 + 25x - 12} \\
\underline{6x^3 + \ 9x^2} \\
22x + 25x \\
\underline{22x + 33x} \\
-8x - 12 \\
\underline{-8x - 12} \\
0
\end{array}
$$

$$
\begin{aligned}
6x^3 + 31x^2 + 25x - 12 &= (2x+3)(3x^2 + (x-4)) \\
&= (2x+3)(3x-1)(x+4)
\end{aligned}
$$

6. **a.** Let $f(x) = x^3 - 3kx^2 + x + 5$.
When the divisor is $x - 2$, the remainder is $f(2) = 9$.

$$
\begin{aligned}
(2)^3 - 3k(2)^2 + 2 + 5 &= 9 \\
8 - 12k + 2 + 5 &= 9 \\
k &= \frac{1}{2} \quad \text{or} \quad 0.5
\end{aligned}
$$

b. Let $f(x) = rx^3 + gx^2 + 4x + 1$.
When the divisor is $x - 1$, the remainder is $f(1) = 12$.

$$
\begin{aligned}
r(1)^3 + g(1)^2 + 4(1) + 1 &= 12 \\
r + g &= 7 \qquad (1)
\end{aligned}
$$

When the divisor is $x + 3$, the remainder is $f(3–) = –20$.

$$
\begin{aligned}
r - 3^3 + g(-3)^2 + 4(-3) + 1 &= -20 \\
-27r + 9g &= -9 \\
3r - g &= 1 \qquad (2)
\end{aligned}
$$

Solving $(1) + (2)$, $\quad 4r = 8$
$\qquad r = 2.$
Substituting into (1), $g = 5.$

Chapter 1 Test

1. **a.**

$$
\begin{aligned}
&18x^2 - 50y^2 \\
&= 2(9x^2 - 25y^2) \\
&= 2(3x - 5y)(3x + 5y)
\end{aligned}
$$

b.

$$
\begin{aligned}
&pm^3 + m^2 + pm + 1 \\
&= m^2(pm + 1) + (pm + 1) \\
&= (pm + 1)(m^2 + 1)
\end{aligned}
$$

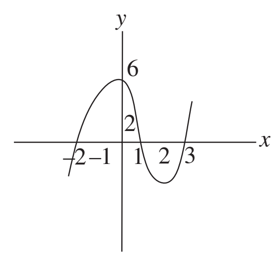
c.

$$
\begin{aligned}
&12x^2 - 26x + 12 \\
&= 2(6x^2 - 13x + 6) \\
&= 2(3x - 2)(2x - 3)
\end{aligned}
$$

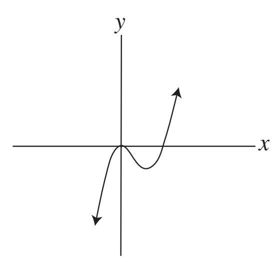
d.

$$
\begin{aligned}
&x^2 + 6y - y^2 - 9 \\
&= x^2 - (y^2 - 6y + 9) \\
&= x^2 - (y - 3)^2 \\
&= (x + y - 3)(x - y + 3)
\end{aligned}
$$

2. **a.** $y = (x + 2)(x - 1)(x - 3)$
The x-intercepts are –2, 1, and 3. The y-intercept is 6.



b. $y = x^2(x - 2)$
The x-intercepts are 0 and 2.
The y-intercept is 0.



3. **a.**

$$\begin{array}{r} x^2 - 7x + 20 \\ x+2\overline{)x^3 - 5x^2 + 6x - 4} \\ \underline{x^3 + 2x^2} \qquad\qquad \\ -7x^2 + 6x \qquad \\ \underline{-7x^2 - 14x} \qquad \\ 20x - 4 \\ \underline{20x + 40} \\ -44 \end{array}$$

The quotient is $q(x) = x^2 - 7x + 20.$

The remainder is $r(x) = -44.$

b.

$$\begin{array}{r} x^2 + 3x + 3 \\ x-3\overline{)x^3 - 6x + 2} \\ \underline{x^3 - 3x^2} \qquad\qquad \\ 3x^2 - 6x \qquad \\ \underline{3x^2 - 9x} \qquad \\ 3x + 2 \\ \underline{3x - 9} \\ 11 \end{array}$$

The quotient is $q(x) = x^2 + 3x + 3.$
The remainder is 11.

4. Since when $f(x)$ is divided by $(x - 1)$, $f(1)$ is the remainder and $f(1) = 0$, then the remainder is 0. When the remainder is 0, the divisor $(x - 1)$ is a factor.

5. Let $f(x) = x^3 - 6x^2 + 5x + 2$. When dividing by $(x + 2)$, the remainder is $f(-2)$.

$$\begin{aligned} r &= f(-2) \\ &= (-2)^3 - 6(-2)^2 + 5(-2) + 2 \\ &= -40 \end{aligned}$$

6. Let $f(x) = x^3 - 3x^2 + 4x + k.$
When $f(x)$ is divided by $(x - 2)$, the remainder is $f(2)$.

$$\begin{aligned} f(2) &= 7 \\ (2)^3 - 3(2)^2 + 4(2) + k &= 7 \\ k &= 3 \end{aligned}$$

7. **a.**

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$
1	–1	0	2
2	–1	2	2
3	1	4	
4	5		

Since the second differences are constant, the points lie on a graph of a quadratic function.

b. Using the graphing calculator, the cubic function is given as $f(x) = 2x^3 - 3x^2 = 5x - 8$. Since for the function $f(1) = -4$, $f(2) = 6$, $f(3) = 34$, and $f(4) = 92$, it is the simplest polynomial function.

8. Let $f(x) = x^3 + cx + d.$

When $f(x)$ is divided by $x - 1$, the remainder is 3.

$$\begin{aligned} f(-1) &= 3 \\ (-1)^3 + c(-1) + d &= 3 \\ -c + d &= 4 \qquad (1) \end{aligned}$$

When $f(x)$ is divided by $x - 2$, the remainder is –3.

$$\begin{aligned} f(-2) &= -3 \\ (-2)^3 + c(-2) + d &= -3 \\ -2c + d &= 5 \qquad (2) \end{aligned}$$

Solving the resulting equation,

(2) – (1) $\quad c = 1$

$\qquad\qquad\quad d = 3.$

9. By dividing $x^3 - 2x^2 - 9x + 18 = (x - 2)(x^2 - 9).$

So, the other factors are $(x - 3)$ and $(x + 3)$.